

Microsoft Word 11.0.6359; 1. Definite Integrals
 The definite integral

$$\int_a^b f(x)dx$$

of a function $f(x)$ defined on the interval $[a, b]$ is given by

$$\int_a^b f(x)dx = \lim_{\|P\| \rightarrow 0} \sum_{i=1}^n f(x_i)\Delta x_i$$

where x_i is a point in the i th subinterval of the partition

$$P = \{a = x_0 < x_1 < x_2 < \dots < x_n = b\}$$

of the interval $[a, b]$,

$$\Delta x_i = x_i - x_{i-1}$$

, and

$$\|P\| = \max \{\Delta x_i\}$$

. The sum

$$\sum_{i=1}^n f(x_i)\Delta x_i$$

is called a Riemann sum. The function f is integrable on $[a, b]$ if the preceding limit exists.

If f is integrable on $[a, b]$, then

$$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{i=1}^n f\left(a + i \cdot \frac{b-a}{n}\right)$$

2. Matrix Equations

The system of equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned}$$

contains the same information as the following matrix equation:

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$